राज्य सेवा (मुख्य) परीक्षा - २०२५ परीक्षा दिनांक ०२ मे, २०२६

2025

C24(B)

BOOKLET NO.
247011

Statistics

Paper – II

Time Allowed : Three Hours **Maximum Marks : 250**

Medium : English **Type of Paper : Conventional / Descriptive**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

1. **There are EIGHT questions divided in two Sections, out of which FIVE are to be attempted.**
2. **Question No. 1 and 5 are compulsory. Out of the remaining, THREE are to be attempted choosing at least ONE question from each Section.**
3. The number of marks carried by a question/sub-question is indicated against it.
4. The medium of answer should be mentioned on the answer book as claimed in the application and printed on admission card. The answers written in medium other than the authorized medium will not be assessed and no marks will be assigned to them.
5. Wherever option has been given, only the required number of responses in the serial order attempted shall be assessed. Unless struck off, attempt of a question shall be counted even if attempted partly. Excess responses shall not be assessed and shall be ignored.
6. Candidates are expected to answer all the sub-questions of a question together. If sub-question of a question is attempted elsewhere (after leaving a few page or after attempting another question) the later sub-question shall be overlooked.
7. Keep in mind the word limit indicated in the question if any.
8. Any page or portion of the page left blank in the Answer Booklet must be clearly struck off.
9. Unless otherwise mentioned, symbol and notation have their usual standard meanings. Assume suitable data, if necessary and indicate the same clearly.
10. Neat sketches may be drawn, wherever required.

Note : Candidates will be allowed to use Scientific (Non-programmable type) calculators.

P.T.O.

SECTION – A

Q1. Solve the following : **(10×5=50)**

Q1. (a) A pharmaceutical company monitors the weight (in gm) of tablets produced by a compression machine. Every hour, a random sample of 5 tablets is taken from 20 preliminary samples, the grand mean of sample means is $\bar{\bar{X}} = 50$ gm and the average range $\bar{R} = 4$ gm. Construct $\bar{X}$-chart and R chart. Interpret the results obtained.
(Given constants for n = 5, $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$) **10**

Q1. (b) Consider a double sampling plan $(n_1, c_1, r_1, n_2, c_2)$. Derive analytical expression for the probability of acceptance and Average Sample Number (ASN) function. **10**

Q1. (c) Solve the following Linear Programming Problem (LPP) using graphical method.

Max. $Z = 4x_1 + 3x_2$

Subject to $3x_1 + 5x_2 \leq 5$

$x_1 - x_2 \leq 0$

$x_1, x_2 \geq 0$ **10**

Q1. (d) Write duals of the following problems.

i) Min $Z = 15x_1 + 12x_2$

Subject to $x_1 + 2x_2 \geq 3$

$2x_1 - 4x_2 \leq 5$

$x_1, x_2 \geq 0$

ii) Min $Z = 5x_1 + 6x_2$

Subject to $x_1 + 2x_2 = 5$

$-x_1 + 5x_2 \geq 3$

x_1 unrestricted

$x_2 \leq 0$. **10**

Q1. (e) How Monte-Carlo method can be used to estimate the value of π(Pi) ? Derive the estimator. Verify its unbiasedness. **10**

Q2. (a) A precision engineering company manufactures steel shafts for automobile gear assemblies. The diameter of each shaft is a critical quality characteristic. Historical data shows that, when the process is operating normally, the shaft diameter is normally distributed with mean $\mu_0 = 50$ mm and standard deviation $\sigma = 1$ mm.
Individual diameters are measured sequentially and the following observations are recorded.

Observation No. :	1	2	3	4	5	6	7	8
Diameter (mm) :	49	50	51	52	51	53	52	54

The engineer wants to detect a shift of 1 standard deviation in process mean.

i) Determine appropriate values of the reference value 'k' and decision interval 'h' for the CUSUM chart. 4

ii) Construct the two sided CUSUM statistics for the given data tabulating C_i^+ and C_i^- for each observation. 8

iii) State the decision rule and identify at which observation, if any, the process is out of statistical control. 4

iv) Explain why a two-sided CUSUM chart is more appropriate than a Shewhart chart in this situation. 4

(Total : 4+8+4+4=20)

Q2. (b) Solve the following Linear Programming Problem (LPP) using Simplex method.

Maximize $Z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \leq 4$

$2x_1 + 2x_2 \leq 5$

$x_1, x_2 \geq 0$ **20**

Q2. (c) Consider a homogeneous discrete time Markov chain with state space $S = \{1, 2, 3\}$ and transition probability matrix.

$$P = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{pmatrix}$$

Find the steady-state distribution of the Markov chain. **10**

Q3. (a) Let $T_1, T_2, \ldots T_n$ denote the lifetimes (in days) of n identical and independent components. Assume that T_i are independent and identically distributed with exponential (λ), $\lambda > 0$. Each component is subject to independent right censoring so that observed data are

$X_i = \min(T_i, C_i)$, $\delta_i = I(T_i \leq C_i)$

i.e. $\delta = 1$ if $T_i \leq C_i$ and 0 if $T_i > C_i$

The censoring times C_i are independent of T_i. The observed data are as follows.

Unit :	1	2	3	4	5	6
X_i :	12	20	25	30	35	40
δ_i :	1	1	0	1	0	1

i) Write the likelihood function for λ.

ii) Derive maximum likelihood estimator of λ.

iii) Compute numerical value of the MLE using the data.

iv) Obtain Fisher information and an appropriate 95% confidence interval for λ.

(Total = 05+05+05+05 = 20)

P.T.O.

Q3. (b) Solve the following Linear Programming Problem (LPP) using Big-M method. Does this problem has feasible solution ?

Maximize $Z = 2x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \geq 6$

$2x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ **20**

Q3. (c) For a Morkov chain define the following :

i) Recurrent state

ii) Transient state

iii) Aperiodic state

iv) Absorbing state

v) Communicating state **(2×5=10)**

Q4. (a) Solve the following payoff matrix game and find the optimal strategies and value of the game. The payoffs are for Player A. **20**

A \ B	B_1	B_2
A_1	3	1
A_2	0	2

Q4. (b) In a Bank, customers arrive at a service counter according to a Poisson process at the rate of $\lambda = 6$ customers per hour. The service time follows an exponential distribution with mean service time of 8 minutes. Assuming M/M/1 queing system, find

i) Traffic intensity

ii) Probability that the system is empty

iii) Average number of customers in the system

iv) Average number of customers in the queue

v) Average waiting time in the system. **(4×5=20)**

Q4. (c) Let T be a non-negative continuous random variable with survival function.

$R(t) = P(T > t)$, $t \geq 0$ and hazard rate function

$$h(t) = \frac{f(t)}{R(t)}$$

i) Define Increasing Failure Rate (IFR) and Decreasing Failure Rate (DFR) classes of lifetime distribution.

ii) Verify whether Weibull distribution belongs to IFR/DFR class. **10**

SECTION – B

Q5. Solve the following : **(10×5=50)**

Q5. (a) Consider a time series $Y_t = \frac{X}{2} + \epsilon_t, -\infty < t < \infty$, where a_t, ε_{t+1} . . . . and X are independent variables with zero mean and unit variance. Derive the auto correlation function of the series. It is a stationary series. **10**

Q5. (b) What is time reversal test ? Verify whether Laspeyre's index satisfy time reversal test. **10**

Q5. (c) Write a note on uses and limitations of index numbers. **10**

Q5. (d) Define the following terms. Also give the formulas of the terms, wherever necessary (for i to v).

i) Fertility and fecundity

ii) Crude birth rate

iii) General fertility rate

iv) Age-specific fertility rate

v) Gross reproduction rate. **(2×5=10)**

Q5. (e) A standardized psychological test has a reliability co-efficient of 0.78 and a standard deviation of 18 in a normative population.

i) Determine the maximum validity coefficient that this test can theoretically attain.

ii) Compute the standard error of measurement for the test and explain its psychometric significance.

iii) The test is subsequently administered to a more homogeneous group in which the standard deviation of scores is 12. Assuming that the Standard Error of Measurement (SEM) remain invariant, estimate the reliability coefficient of the test in this group.

iv) Decompose the observed score variance into true score variance and error variance and state the population of variance attributable to true scores.

(02+03+03+02 = 10)

Q6. (a) The following table gives quarterly sales (in units) of a product for two years.

Year	Q_1	Q_2	Q_3	Q_4
2022	120	140	180	160
2023	130	150	190	170

Using multiplicative model, obtain the seasonal indices. **20**

Q6. (b) Solve the following problem and interpret results.

i) The following data relate to commodities sold at the wholesale level in a base year and a current year using Laspeyre's method. Compute the Wholesale Price Index (WPI) for the current year, taking base year index as 100.

Commodity	Base Price (p_0)	Base quantity (q_0)	Current price (p_1)
A	20	50	25
B	10	80	12
C	15	40	18

ii) Using the following information, calculate the Consumer Price Index (CPI) by the family budget method (Weighted average of price relative).

Group	Weight (%)	Price Relative
Food	40	130
Clothing	20	120
Housing	25	110
Fuel and Light	15	140

iii) Calculate Agricultural Production Index (API) for the following data.

Crop	Q_0	Q_1	P_0
Wheat	100	120	10
Rice	80	100	12

Where Q_0 is quantity in base year

Q_1 is quantity in current year

P_0 is base year price.

iv) Calculate index of industrial production for the following data.

Industry	q_0	q_1	Weight (W_t)
Steel	50	60	5
Cement	40	44	3

Where q_0 : base year output

q_1 : current year output

w_t : weights

(5×4=20)

Q6. (c) Explain the concepts of percentile scores and Intelligence Quotient (IQ).

i) Suppose 40 students appeared for an exam. A student scored 85 marks and 30 students scored below 85. What is the Student's percentile rank ?

ii) Suppose a child scores 85 marks in a test having mean 70 and standard deviation 10. Calculate IQ of the child. **(06+02+02=10)**

Q7. (a) Consider the general linear model

$\underline{Y} = X\underline{\beta} + \underline{\in}$, $\underline{\in} \sim N(0, \sigma^2 I)$ where $\underline{Y} = (5, 7, 6, 8)'$, $X = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$

i) Obtain the least squares estimate of the parameter vector $\underline{\beta}$.

ii) Compute fitted values and residuals.

iii) Obtain unbiased estimate of σ^2.

iv) Test the hypothesis $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$ at 5% level of significance.

(Given $t_{0.025, 2} = 4.303$) **(06+04+04+06=20)**

Q7. (b) i) Explain the concept of population growth curve. State the assumptions under which a population is said to follow the exponential law of growth. Derive the corresponding differential equation and solve it to obtain population function.

ii) Formulate the logistic growth differential equation by incorporating the idea of limited resources.

iii) Assuming the population follow exponential growth law with growth rate $r = 0.05$ per year. Find the population after 10 years.

(Assume initial population is 100) $e^{0.5} = 1.6487$

iv) Assuming that the population follow the logistic growth model with growth rate $r = 0.2$ per year and carrying capacity $k = 1000$. Find the population after 10 years.

(Initial population is 100), $e^{-2} = 0.135$. **(08+08+02+02=20)**

Q7. (c) Define generalized least squares. State the assumptions of generalized least squares. Derive generalized least squares estimator $\hat{\beta}_{GLS}$ for the general linear model. $\underline{Y} = X\beta + \underline{\varepsilon}$, $V(\underline{\varepsilon}) = \Omega$. **10**

Q8. (a) Answer the following :

i) Give meaning and importance of official statistics.

ii) Describe the organizational structure of the official statistics in India.

iii) Explain major sources of official statistics in India.

iv) Discuss the challenges faced by official statistics system in India.

(05+05+05+05=20)

Q8. (b) For the following data related to a population of women in the reproductive age group during 2024.

Age group of Women (Years)	No. of Women	No. of live births
15 – 19	12000	480
20 – 24	10000	1600
25 – 29	9000	1980
30 – 34	8000	1200
35 – 39	7000	560
40 – 44	6000	180
45 – 49	5000	40

Total population = 1,20,000

Total no. of live births during the year = 6040.

i) Calculate Age-specific fertility rate for each group (per 1000 women).

ii) Calculate General Fertility Rate (GFR).

iii) Calculate Crude Birth Rate (CBR).

iv) Calculate Total Fertility Rate (TFR).

v) Interpret the TFR, comment on the fertility pattern of the population.

(04+04+04+04+04=20)

Q8. (c) What is multicollinearity in multiple regression model ? Discuss its consequences and suitable remedies. **10**